THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR 3/19

FROM 'UH-OH' TO 'AH-HA!'

INSIDE

Greg Oates - Senior lecturer, mathematics education, University of Tasmania

FOSTERING INTUITION IN THE CLASSROOM

Inquiry-based learning; problem solving; challenging tasks; student-centred learning; active learning: These terms are commonly used for an approach to learning mathematics which sees the student develop their own understanding as they work through a mathematical problem, and such approaches are - quite rightly - widely promoted in our classrooms.

These approaches give students better chances to experience authentic mathematical experiences as they attempt to model real life problems and data. The exact nature of the inquiry can vary greatly, from a more traditional confirmation inquiry where the teacher frames questions for the student, usually with a specific pre-determined outcome, on to structured and guided inquiry with decreasing levels of scaffolding, through to open inquiry, where students formulate their own research question(s).

Continued on page 4

THE COMMON DENOMINATOR
The MAV's magazine published for its members.

Magazine 272, July 2019

The Mathematical Association of Victoria, 61 Blyth Street Brunswick VIC 3056

ABN: 34 004 892 755
Tel: 03 9380 2399
Fax: 03 9389 0399

office@mav.vic.edu.au
www.mav.vic.edu.au

Office hours:
8.30am – 5pm
Monday to Friday

President: Michael O'Connor
CEO: Peter Saffin

The Common Denominator is edited and produced by Louise Gray, Stitch Marketing.

Print Post Approved
Publication No: PP100002988

FROM THE PRESIDENT

Michael O'Connor

Midwinter. Cold. Dark. Coughs. Colds. Marking. Reports.

As you read this I expect many of you have just returned from the well earned break and are about to begin again. The dark, the cold and the wet will have taken their toll.

The Winter Solstice, though, has been throughout human history and before a time for celebration and for looking forward to longer days and the promise of new growth. So important was it to our ancestors that they built enormous structures to mark its dawning each year.

I recently had the privilege of visiting Newgrange in Meath County Ireland. Constructed by farming communities over 5000 years ago it is a marvel of engineering, observational astronomy and, of course, mathematics. Long before watches or online calendars these people understood the importance of accurate measurement and calculation. Their lives, in an absolute sense, depended on it. Ours do as well, but the directness of the connection is no longer as clear.

Specialisation has, over millennia, made it harder to discern what is needed to be numerate in a high tech society. Geography, commerce and science are all underpinned by mathematics. Ian Stewart, an English mathematician and writer, once suggested that we should have a 'Maths Inside' sticker in the same way as Intel used to have 'Intel Inside'.

In the absence of such stickers we have the community of peers that is MAV. As classroom teachers it can, from time to time, feel like we are toiling away on our own, in the dark. Particularly at midwinter. The promise of solstice however is that new growth will come again. Ideas, like sunlight, are absorbed and transformed from the world around us into solid structures bound together by the trunk and branches. Every root and leaf benefits as long as it remains connected to the trunk. The MAV is the trunk of our tree. All of the services offered, the publications, the conferences, MAVshop and professional development are for the benefit of us, the members.

In addition to the formal services MAV provides, there are the connections between members. We have representatives from early childhood, primary and secondary schools, universities and business and industry. I encourage everyone to make the most of what is available to you. Curiosity and enthusiasm are at the root of all learning. Allowing our students to see and share in our passion enkindles their own. A recent article in EduResearch Matters from AARE says that students notice and value the expert knowledge their teachers bring to the classroom. They also know the passion and enthusiasm of their teachers to be infectious. May this be the only infection you pass on to anyone this year.

REFERENCES

Letters to a Young Mathematician, Ian Stewart, 2006, Basic Books

AARE: EduResearch Matters
www.aare.edu.au/blog/?p=3410

VALE GRAHAM WILLIS

Graham Willis, 1981.

All at MAV were saddened to learn of the passing of Honorary Life Member, Graham Willis, in May 2019. Graham was a driving force in Victorian mathematics education and the revitalisation of MAV during the 1970s. Graham's legacy will live on in many classrooms - he invented the *Trigmaster*, a tool to demystify trigonometry which many teachers still swear by. A longer piece detailing Graham's life and passion for mathematics education has been published in MAV's secondary journal, *Vinculum* (Term 3 edition, 2019).

MAV PROFESSIONAL DEVELOPMENT

During Term 3 2019, a variety of presenters and MAV's own mathematics educational consultants will present workshops focussing on innovative teaching practice.

Make sure you reserve a place by booking online early, www.mav.vic.edu.au/pd.

TOPIC	DATE	YEARS	PRESENTER
Horsham P - 10 Mathematics education conference	2/8/19	F - 10	Various
Using Excel for basic coding	7/8/19	8 - 10	Danijela Draskovic and Helen Haralambous
Making connections between problem solving and mathematical proficiency	8/8/19	F - 6	Paul Staniscia
Using learning walls	18/9/19	F - 6	Cathy Epstein
Introduction to coding with the TI-Nspire	21/8/19	7 - 10	Danijela Draskovic
Maths300: How to use (website and software)? Why?	3/9/19	F - 6	Marissa Cashmore
More warm ups	6/9/19	7 - 9	Helen Haralambous

VCE STUDENT REVISION LECTURES

Each year The Mathematical Association of Victoria presents VCE revision lectures for students in the three VCE Mathematics studies: Mathematical Methods, Specialist Mathematics and Further Mathematics.

All lectures are delivered by highly qualified, experienced teachers who are current VCAA exam assessors. This means that the information the students receive is the most relevant and up-to-date, and the tips will be based on where students typically make mistakes in their VCE exams.

As well as the lecture, students will also receive a detailed and comprehensive booklet of revision notes that has been developed by exam assessors.

Students benefit greatly from this unique experience. The timing of the lectures in the September/October school holiday period is a great time for students to revise; not too far from their looming exams, but also not too close to that hectic pre-exam period.

Due to popular demand, MAV has introduced two new venues this year: Ringwood and Broadmeadows. Places are limited, so don't miss out on your chance to attend. Book via www.mav.vic.edu.au. Teachers note: Group booking discounts available! Contact Danijela Draskovic, ddraskovic@mav.vic.edu.au.

REGION	WHERE	WHEN
Eastern metro	Glen Waverley	Sunday 22 September
South East metro	Mt Eliza	Monday 23 September
Western metro	Taylors Lakes	Tuesday 24 September
South East	Sale	Wednesday 25 September
South West	Horsham	Monday 30 September
Northern metro	Broadmeadows	Tuesday 1 October
North East	Wangaratta	Saturday 5 October
Eastern metro	Ringwood	Sunday 13 October

Feedback from past participants:

'Thank you for saving me. The lecture was very detailed. It was definitely well worth the time, effort and money!'

'Lecture went through aspects we hadn't covered in class.'

'Really well presented and structured lecture with an engaging intelligent lecturer.'

'Really, really, really good!!'

FROM 'UH-OH' TO 'AH-HA!'

Greg Oates - Senior lecturer, mathematics education, University of Tasmania

CONT. FROM PAGE 1.

Figure 1. Starting cards: What could the next card be? Have a guess.

Figure 2. Now, what could the next card be? Make a prediction.

Figure 3. Second turn over: Were you right? What might the next card be now? Are you sure?

Figure 4. Third turn over: Do we know what is next now?

There are many rich examples of inquiry based learning resources in mathematics available for teachers and students, with varying levels of scaffolding. In the Australian context, there are, among others, reSolve, Maths300, and Maths Inside (all supported by AAMT).

The reSolve protocol makes explicit the importance of providing purposeful, inclusive and challenging tasks within a knowledge building culture which celebrates productive struggle and the confidence to take risks (http://resolve.edu.au/protocol).

Most teachers and resource developers would agree however that student's ability to engage with inquiry learning often requires them to take risks and to build on initial intuitions they may have about the problem.

The best problems are thus usually seen as being accessible to a range of abilities and levels, what is known as 'low floor, high-ceiling' tasks. But even a low floor does not always allow students to get started on the problem. Students can frequently doubt their intuitions and be afraid to follow them for fear of being wrong. We need to find ways of encouraging students to think intuitively, to have a productive disposition towards trying things out.

Following your intuitions down sometimes blind alleys is a natural part of authentic mathematical enquiry (think for example of Andrew Wile's path to proving Fermat's last theorem, as immortalised in the book and film by Simon Singh). There is a big difference between a 'wrong guess or thought', and misconceptions (often ingrained) and mathematical errors in say calculations or incorrectly applied procedures. Sadly, I feel students often see these collected under the same umbrella of 'wrong' and marked with a 'X'.

I try to build 'safe' classrooms, where students are not afraid to be wrong, where its safe to take risks, exactly because being wrong is often what leads to real learning. I believe that we should start this early and start out simple, but also that its never too late to start. A simple example I have used at many levels of the curriculum is a 'pattern-cards' game, where I first show students examples of a series of cards that indicate a pattern, but in the early stages at least the pattern they detect might be (deliberately) the wrong one. I do not claim that the game is itself original, I have seen many such examples elsewhere. However, what I do wish to highlight is the intent behind the approach. I use it to deliberately provoke intuitions which may be wrong, as much as addressing the algebra and patterns content strand within which this game is likely positioned.

PATTERN CARDS

There are of course infinite possibilities for the first two turnovers (see images above), as indeed there are for the third, although the latter is usually less obvious to students, wherein lies the opportunity for surprise: 8 is probably the most common suggestion for the third turnover, but it could easily be shown as 5 (repeating growing pattern of +1, +2, +1, +2..) or many other possibilities.

The question of whether students are then sure what the pattern is after the third turnover is interesting. Most people will say '10' for the next card (which is what I will turn over) and most will also be convinced that this is now the only possibility, so I leave this as a challenge to see if they can come up with an alternative that works.

The process I promote to the students is:

- Say what you think (or believe), make a prediction
- Convince yourself, test your answer
- Convince a friend or your teacher your answer is correct
- Can you prove it?

Once we have tried a few as a group, I usually encourage the students to design their own patterns, and see if they can fool their friends. The challenge is who can devise a pattern that needs the biggest number of cards before their partner gets the pattern. The level at which you use this game can be varied, by varying the conceptual complexity of the patterns, for example with simple repeating patterns in the early years; introducing the Fibonacci Sequence in the middle years, or geometric and arithmetic sequences in later years, or the degree to which you develop the structure of the pattern, in for example a verbal description such as doubling in lower years to formulating algebraic rules at higher levels.

Psychologically, questions or problems that lead students to intuitively wrong answers can be viewed as setting up cognitive dissonance, which can be a powerful motivator for change, if treated in a productive manner (cognitive dissonance is also viewed as somewhat traumatic, which reinforces the need for care in developing a safe classroom).

I first started collecting such problems after trying out two problems in a university foundation mathematics (transition) course in 1995. The problems I used then came from a 1991 article I found by Avital & Barbeau called 'Intuitively misconceived solutions to problems' (*For the Learning of Mathematics*, *11*(3)). They provoked intense reactions from my students, and ultimately led to my thinking about 'safe classrooms'.

There are many famous problems in the history of mathematics which testify to the frequent stumbling path of mathematical discovery, but which have ultimately become turning points in our mathematical knowledge, e.g the Königsberg Bridge problem which led to Euler's development of graph theory in 1736 (see the TEDEd talk *How the Königsberg bridge problem changed mathematics* by Dan Van der Vieren: www.youtube.com/watch?v=nZwSo4vfw6c). It is important that we give students a sense that this actually the way mathematics works, and provide opportunities for them to experience it for themselves.

For me there is nothing better than those 'Ah-ha' moments in the classroom. It's those moments when we see students' faces light up at the pleasure of solving a problem they have been struggling with. And there is nothing sadder than students' being too scared to try.

I encourage teachers to start the process of building safe classrooms early on, using tasks that are deliberately designed to go against students' intuitions or beliefs about mathematics they may previously have constructed, but with care to always make it clear to your students that being wrong can also be the right way to learn!

Dr Greg Oates is a keynote speaker at MAV's annual conference in December. MAV19 is the premier event for mathematics educators in Australia. At the heart of MAV's Annual Conference are teachers. Each year over 1400 mathematics educators including teachers, academics, policy makers, curriculum experts and resource developers come together to share their expertise, experiences and ideas. Register for the conference now at www.mav.vic.edu.au.

UNPACKING DYSCALCULIA

Nathalie Parry - Sessional Lecturer, Melbourne Graduate School of Education, University of Melbourne

For some, mathematics is a wonderful subject. It is logical. It makes sense. For some students, mathematics is anything but. They are faced with added challenges in mathematics due to learning disabilities.

A learning disability is a disorder that inhibits the ability to process and retain information. These processing problems can interfere with learning basic skills such as reading, writing and/or maths and also interfere with higher level skills such as organisation, time, planning, thinking, long or short term memory and attention, interestingly, all skills needed in mathematics.

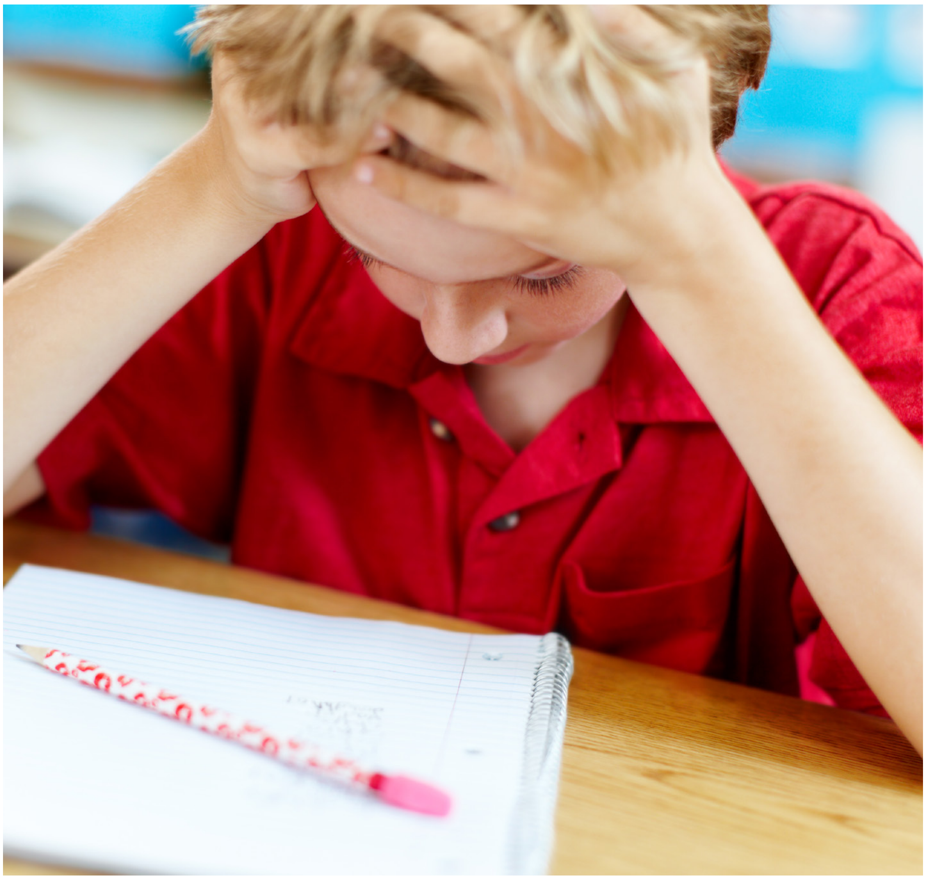
People with learning disabilities are generally of average or above average intelligence but with gaps between their potential and actual achievement. This is why learning disabilities are referred to as 'hidden disabilities': the person seems to be very bright and smart, yet may be unable to demonstrate the skill level expected from someone of a similar age.

Problems in numeracy are thought to be as widespread as literacy difficulties; however, there has been more research on dyslexia than dyscalculia (Butterworth, 2004).

Dyscalculia is a condition that affects the ability to acquire mathematical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers and have problems learning number facts and procedures. Even if they produce a correct answer, or use a correct method, they may do so mechanically and without confidence. (The National Numeracy Strategy (DfES, 2001))

Dyscalculia is an inherited neurological condition estimated to affect the acquisition of skills in mathematics for about 3-7% of the population (depending on the criteria defined by researchers…) However, the co-existence with other learning disabilities may be as high as 40%. This means that while a student is suspected or diagnosed with a learning disability like dyslexia, or ADHD they will most likely have dyscalculia even if this is not officially diagnosed.

Every student's profile will be different, but students will typically have difficulties with:



- organising objects and sets of items in a logical way
- learning to count, students may use immature strategies to calculate such as counting by ones (difficulty skip counting), often with their fingers
- subitising, determining which number is larger
- recognising number symbols i.e. printed digits or confusing +, -, x, ÷
- understanding mathematical operations and performing calculations
- learning and recalling basic mathematical facts, particularly the times tables
- recognising patterns in numbers
- decomposing numbers
- understanding the structure of numbers such as place value and grouping
- telling the time, perception of the passage of time and difficulties sticking to a schedule
- reading and interpreting graphs, charts and maps
- measurement and understanding spatial relationships/direction
- finding more than one way to solve a maths problem
- grasping abstract concepts like multi-step algorithms, fractions and algebra
- applying mathematical concepts to everyday life, such as budgeting and time management skills.

These challenges can result in increased anxiety and negative attitude towards maths. We can support students with learning difficulties by appealing to their strengths rather then the areas the find most challenging. Manipulatives and diagrams function as cognitive tools to connect students to concepts: they may make difficult ideas understandable, complex problems solvable and abstract concepts tangible by enabling the student to 'act out' or visually represent the given problem.

Concrete materials and manipulatives (Cuisenaire rods, MAB blocks, counters) can assist students in developing the concept of number or learn basic operations such as addition and subtraction. Students should gradually transition towards using diagrams and pictures in place of concrete materials, and then move towards using symbols to represent number work in a more abstract way. Do not remove concrete materials too soon they will help to develop the student's understanding.

Consider for example using the 'bar method' popular in Singapore. The bar model allows students to draw and visualise mathematical concepts to solve problems. Some examples and activities can also be found on the reSolve website.

Introduce and pre-teach new vocabulary. The language used in maths can have different meanings. Students need a comprehensive vocabulary to understand the precise meaning of mathematical terms. Use simple, clear and concise language. Focus on the important content and break complex skills into small manageable steps.

Play games with dice and dominoes so that students can recognise common dot patterns. Give multiplication grids and number bonds to reduce the stress of having to remember these facts and enable to access higher order maths concepts and skills.

Repetition and an 'over-learning' approach will help. Practice will increase fluency in processing, improve retention of information, facilitate recall and develop understanding. All students require many presentations before remembering and learning a new skills. Students with learning difficulties will need significantly more practice. Rhonda Fakota's *Elementary Maths Mastery* series is highly recommended.

Give students more time. As already mentioned specific learning difficulties are processing difficulties. Being quick at maths does not guarantee success. However working memory capacity is further reduced by anxiety and fear of failure. As you can imagine, repeated failure or setbacks in maths can be defeating and demotivating. Students with low self-efficacy are reluctant to engage in tasks where those skills are required, and if they do, they are more likely to give up when they first encounter difficulty.

Instead build students self efficacy by ensuring all students experience levels of success. A positive academic self-concept has a positive impact on achievement (Hattie 2009) resulting in better learning opportunities and improved outcomes.

For more information on how to support students with math learning difficulties refer to:

- AUSPELD Understanding Learning Difficulties A practical guide (available via Scootle)
- Speld Victoria www.speldvic.org.au
- Learning Difficulties Australia www.ldaustralia.org
- Steven Chinns Books: The Trouble with Maths, and More Trouble with Maths
- Ronit Bird www.ronitbird.com

THE HISTORY OF MATHEMATICS

Terence Mills

We often associate ancient Egypt with pyramids, sphynxes, the Nile, Cleopatra, and Moses. However, the mathematics of ancient Egypt is also particularly fascinating, and has received considerable attention from researchers since the early 20th century.

An important source of our knowledge about mathematics in this era comes from the Rhind mathematical papyrus (Peet 1923). The papyrus is at least 3000 years old, and is likely to be a copy of an even older document. It is known as the Rhind papyrus because it was purchased by a Scottish archaeologist Alexander Henry Rhind (1833-1863) in Luxor, Egypt in 1858, and eventually found its way to the British Museum after his death.

The papyrus is essentially a set of solved mathematical problems, most of which are put in the context of some application to geometry, weights and measures, or financial transactions.

However, it does not set out proofs of general statements as we see in Euclid's Elements.

Particularly interesting is the treatment of fractions. The Egyptians had notations only for two types of fractions, namely $\frac{1}{n}$ where n is a positive integer (or unit fractions) and $\frac{2}{3}$ which was exceptional. Other fractions were expressed as sums of two, or more, distinct unit fractions.

The first entry in the Rhind mathematical papyrus is a table of expressions for fractions of the form $\frac{2}{n}$ where $n = 5, 7, 9, \ldots, 101$. There is no need to consider even values of n because we can always reduce such fractions; e.g. $\frac{2}{30} = \frac{1}{15}$. The unit fractions must be distinct. So, for example, $\frac{2}{5} = \left(\frac{1}{3}\right) + \left(\frac{1}{15}\right)$.

If you try a few yourself, you will see how difficult these problems can become. Try $\frac{2}{13}$.

We might wonder, how did the Egyptians effect these calculations?

REFERENCES

Peet, T. E. (1923) *The Rhind mathematical papyrus*. Hodder & Stoughton Limited, London.

CLASSPAD AND MATHS METHODS

Alastair Lupton - Coordinator of the Sciences, Le Fevre High School

GET THE MOST OUT OF YOUR CLASSPAD IN MATHS METHODS

The ClassPad is a powerful tool for doing mathematics in range of ways. In the context of VCE Mathematical Methods, there are some functions and features that are of particular use.

When doing calculus, it is common to calculate the turning points of a function. This can be done very efficiently using the **fMax** (or **fMin**) commands. These commands, found under the **Interactive-Calculate** menu (Figure 1), will find global maxima or minima, or local maxima or minima if a restricted domain is entered. Exact answers are provided, or the commands can be executed numerically if exact answers are not available or required. Once a calculation is completed, the command line can be edited, to look for a max instead of a min for example, by just changing 2 characters. This tip is performed using the **abc** keyboard (Figure 2).

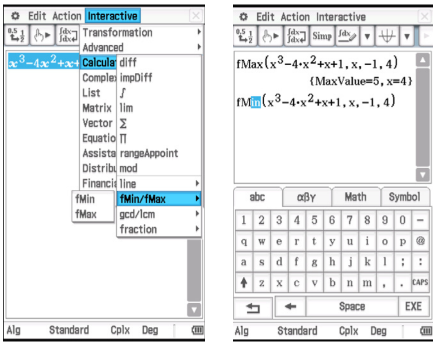


Figure 1 and Figure 2.

When working with a function and making repeated calculations, it is often useful to **define** the function. This command can be found under the **Interactive** menu. It is important to remember that if you define a function as $f(x)$, it will replace any function that was previously defined as $f(x)$. As such, in an exam with a number of questions that use $f(x)$, it may be worth defining a function as $f1(x)$ when completing question 1. That way, the question can be revisited even after later questions have been completed.

Another way to re-use content that has already been entered in to your ClassPad is to 'copy and paste', much as you would on a computer. Select content and then 'tap-drag-drop' into a new working line. Alternatively, the commands **Copy** and **Paste** can be found under the **Edit** menu, just like in many computer software applications. This can be very useful when complex structures have been entered and only minor changes are required to perform additional computations, as is commonplace in examinations. Copy and paste works between apps, so, for example, a function entered in the Main app can be copied into the Graph and Table app.

When performing calculations involving statistical distributions, the **Distribution/Inv.Dist** menu, under Interactive in the Main app, is a time saving alternative to switching to the Statistics app (Figure 3).

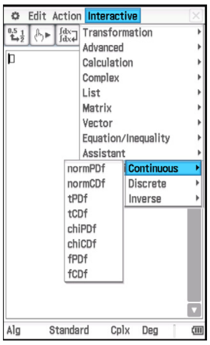


Figure 3.

Once a calculation is entered, it can be edited for further use. This is handy when performing a number of similar calculations, or when solving for an unknown parameter, like finding μ or σ when working with the normal distribution (Figure 4).

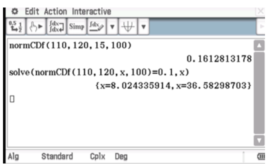


Figure 4.

If there is ClassPad functionality that you need support with, there is an extensive library of over 160 'How to...' videos available at www.casio.edu.shriro.com.au/app/classpad_how_to_videos.php. These videos are sorted by complexity, topic and the VCE course to which they relate. They are very likely to have covered the functionality you need.

In preparation for the examination, knowing when and how to use CAS technology is key to a good result in Mathematical Methods Examination 2. One way to hone your skills in this area is to study ClassPad-enabled solutions to past examinations.

Your school may have solutions to past VCAA exams that feature ClassPad screenshots. Alternatively, solutions to the last eight Math Methods Exam 2's are available at the VCE Study Resources page (under the Classroom Resources tab) at www.casio.edu.shriro.com.au/app/view_module_collection.php?id=5

For the last three examinations, including 2018, these solutions were presented in video format, with 'by hand' alongside ClassPad-enabled solutions, accompanied by a discussion of the methods.

CUSTOMISING YOUR CLASSPAD

Whilst your ClassPad has an encyclopedic list of programmed functionalities, these functionalities can be added to, to make it an even more useful tool. One of the most powerful customisations is the creation of user defined functions (UDFs). There are many aspects of mathematics where we routinely make substitutions into complex formulae, like the distance formula, or undertake multi-step algorithms based on specific input, like finding the equation of a line through two points, and these can often be done very effectively via UDFs.

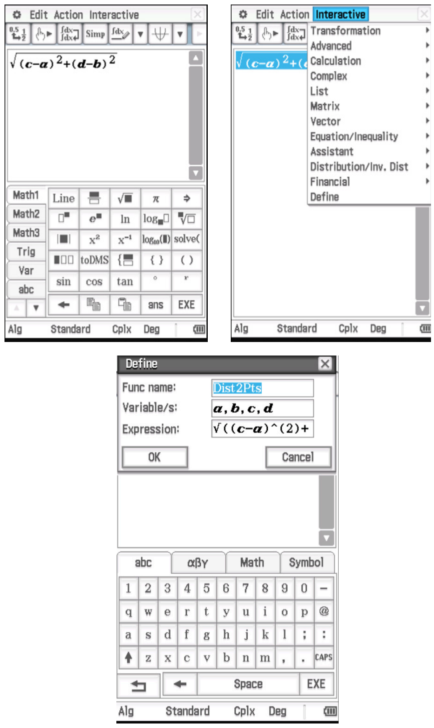


Figure 5 .

In Question 1 of Section B of the 2017 Methods examination, the formula for the distance between two points was used to perform three computations. These computations could be done quickly, and with limited opportunity for error, using a UDF created prior to the exam.

To create a UDF for the distance formula, enter it in terms of 4 distinct variables, representing the coordinates of the two points under consideration i.e. (a, b) and (c, d). Select the formula and then choose Define from the Interactive menu. In the Define window, enter the variables that are being used and give your UDF a memorable name. See Figure 5.

The function can be called up by typing its name, or it can be selected and inputted from the catalog. The catalog, which contains all of the ClassPad's in-built functions as well as all UDFs, can be accessed via the down arrow on the keyboard. The drop down on the right of the catalog can be used to view only User (UDFs). From there, the desired function can be chosen and inputted (Figure 6).

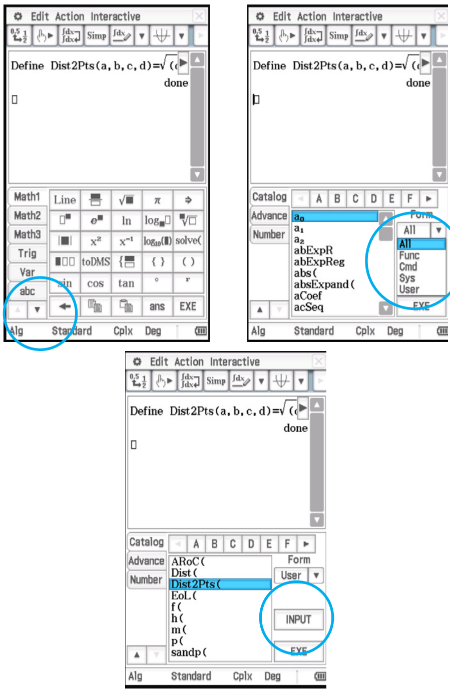


Figure 6.

The computations required for Question 1 were the distance between (–1, 4) and (1, –4), the distance between (–1, –1 + k) and (1, 1 – k) and the value of k for which this distance is equal to k + 1. Figure 7 shows these results, computed using the UDF.

When using UDFs, it is important that they be created prior to the examination.

In this way, valuable time can be saved in the exam and, importantly, the UDFs can be carefully checked to make sure they are functioning correctly. If the above question were presented in an examination and no UDF existed, direct use of the distance formula, along with some clever use of copy and paste, would have been the most efficient way to obtain the required results.

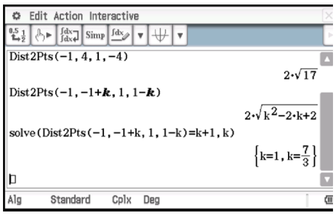


Figure 7.

A video showing in more detail how to create, use and delete UDFs can be found on the VCE Study Resources page, as part of the 'ClassPad Top Tips for Exams – Collection 1' at www.casio.edu.shriro.com.au/app/view_module.php?id=342.

Another way that the utility of your ClassPad can be enhanced is via the creation of spreadsheets to perform calculations upon sets of data. For example, a useful spreadsheet that can easily be created is one that computes the mean, variance and standard deviation of a discrete random variable.

In the spreadsheet shown in Figure 8, columns A and B require the entry of outcomes x and the probabilities $Pr(X = x)$. Column C computes the product $x \times Pr(X = x)$ and column D computes the difference $x - \mu$. Column E computes the product of columns C and D and row 16 does the summing necessary to obtain the mean, variance and standard deviation of the discrete probability distribution.

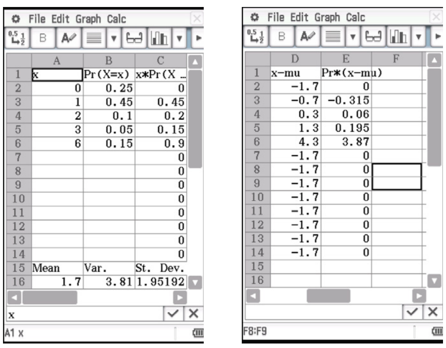


Figure 8.

As it is likely that these sorts of spreadsheets might be required at different times during the course, it is important to know how to save them. Save can be found in the File dropdown menu, and an appropriate name can be entered. Once a spreadsheet is saved, it can be opened at any time via the File menu (Figure 9).

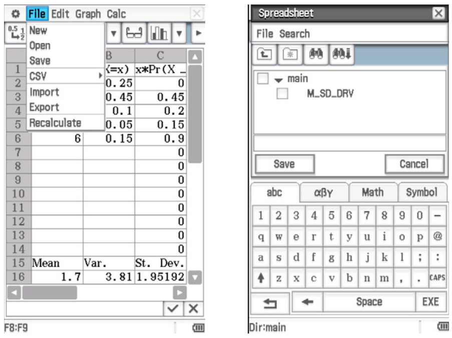


Figure 9.

It is important to create spreadsheets prior to examinations so that time is saved and their correct functioning can be checked. All spreadsheets will have limitations. The one above can only operate upon a discrete random variable with 14 or fewer outcomes, and fractional probability values need to be preceded by an equals sign i.e. $=\frac{3}{20}$.

As long as these sorts of limitations are understood by the user, they can prove to be a very useful tool indeed. Question 12 in Section A of the 2018 Methods examination was an opportunity to use this sort of spreadsheet to quickly obtain a result, in particular the mean shown above.

Another way to customise your ClassPad is by using the eActivity app. eActivities are files created within your ClassPad that can contain the content from any other app, Main, Spreadsheet, NumSolve, Geometry etc, as well as text. For more information visit casio.edu.shriro.com.au.

With strategic use, the ClassPad is a most efficient and powerful tool when undertaking the VCE Mathematical Methods Examination 2. Good use of your ClassPad is sure to save valuable time in this exam. Coupled with a sound knowledge of the curriculum and a good grasp of 'by-hand' algorithms, discerning ClassPad use means that this challenging task can be tackled with confidence.

TI-NSPIRE AND MATHS METHODS

Peter Flynn - Texas Instruments

USING TI-NSPIRE CX CAS WIDGETS AND NOTES TO OPTIMISE EXAM RESULTS AND ENHANCE LEARNING

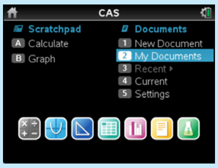
The TI-Nspire CX CAS Notes application is an interactive environment that can be used by teachers and students to perform mathematical calculations with accuracy and efficiency and by teachers to provide rich, collaborative and dynamic mathematical learning experiences. This interactivity originates predominantly from the facility to embed linked mathematical expression boxes (known as Math Boxes) within a text document. These mathematical expression boxes can aid in automating a wide bandwidth of numeric and symbolic calculations and are particularly effective when answering examination questions.

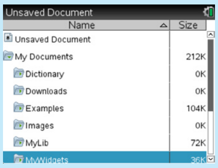
Here I share this little-known interactivity through examination-style examples taken from calculus and vectors (Specialist Mathematics) and offer a brief classroom vignette from algebra and functions, namely the role of the discriminant in classifying quadratics.

For answering examination questions requiring numeric and symbolic calculations to be performed accurately and efficiently, a TI-Nspire CX CAS offers the powerful facility to create, save and use a one page document known as a Widget.

TI-WIDGETS

With TI-Nspire CX CAS, all work created and saved with TI-Nspire CX CAS applications are stored as a document which can be shared with others. In particular, a Widget is a TI-Nspire (.tns) document that is stored and accessed in a user's MyWidgets folder which can be located as shown in the following screenshots.



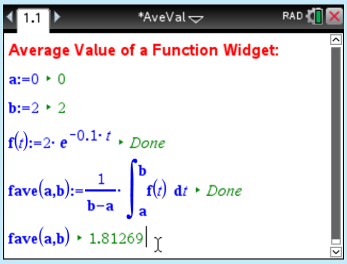


Note that a document is only regarded as a Widget when it is saved or copied to the designated MyWidgets folder. Widgets can be used to seamlessly access text files, insert and run pre-prepared templates that automatically answer suitable examination questions and insert a saved problem into a document. It is important to note that when a Widget is added, TI-Nspire CX CAS extracts only the first page of the selected TI-Nspire (.tns) file and inserts it into the open document. Hence it is advantageous for a Widget to form a lean design and consist of a single page only.

Prior to sitting a CAS-permitted mathematics examination, teachers and students have the opportunity to devise interactive Widgets with embedded mathematical expression boxes that automate standard mathematical calculations and hence readily answer heavily-templated examination questions. Examples of symbolic and numerical calculations suitable for a Widget that are frequently assessed in CAS-permitted examinations include applications of differential and integral calculus, complex number and vector calculations, sequences and series and matrix calculations and the use of the sine and cosine rules to solve triangles.

CREATING, SAVING AND ADDING A TI-WIDGET

Let's say you wish to create a Widget for an upcoming CAS-permitted examination that calculates the average value of a function. After adding a TI-Nspire CX CAS Notes application, set up the Widget as shown here:

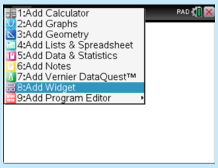


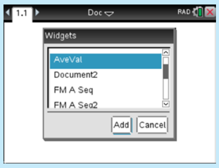
Note that to insert mathematical expression boxes press **Menu** > **Insert** > **Math Box**. Note also the use of the assign command (:=). The input is displayed in blue and the output is displayed in green. **Menu** > **Math Box Options** > **Math Box Attributes** allow the user to change the settings and appearance of a mathematical expression box. Surrounding explanatory and prompting text can also be added to a Widget to provide enhanced understanding of what the Widget is doing or offer advice on how it should be used. For example, to obtain a floating point decimal answer form rather than an exact answer form.

To save the Widget to the MyWidgets folder, navigate to **My Documents** > **MyWidgets**, type in a name for the Widget and save. If an examination question asks candidates to calculate the average value of a function, access the Widget in one of the two following ways.

1. To add a Widget to a New Document

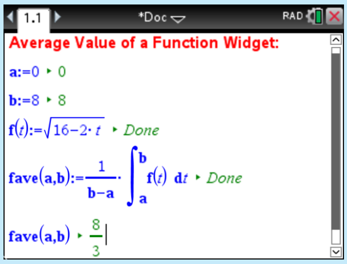
Open a **New Document**, click **Add Widget**, scroll to select a .tns file from the box and click **Add**.





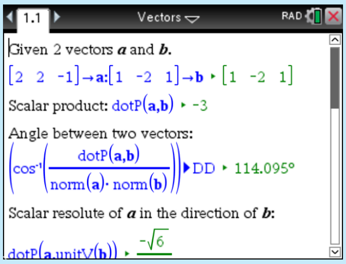
2. To add a Widget to an existing document

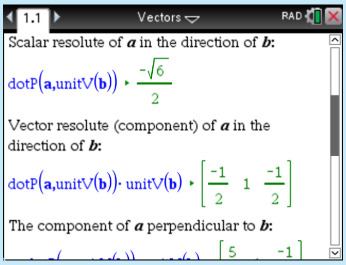
Click **Doc** > **Insert** > **Widget**. Now all the student has to do is to correctly interpret the exam question and fill out the mathematical expression boxes requiring alteration (remember to press **Enter** each time). If the exam question is worth more than one mark, it is important to show appropriate working. Students must also be cognisant of answer form if one is specified. The screenshot shows the Widget used to calculate the average value of another function.



VECTOR CALCULATIONS: FUNCTIONAL USE

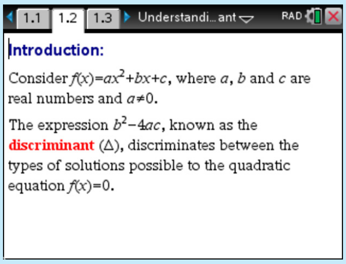
The two screenshots on page 11 show a Widget called Vectors. Displayed on the Widget is some explanatory text and various vector calculations for two stored vectors ***a*** and ***b***. As discussed earlier, once such a Widget is set up and saved, students can access it from their MyWidgets folder and edit the two vectors to obtain an updated set of results.

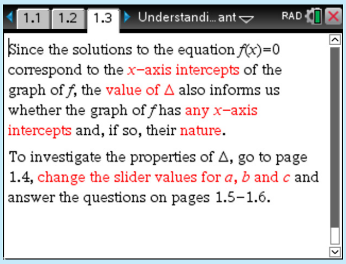




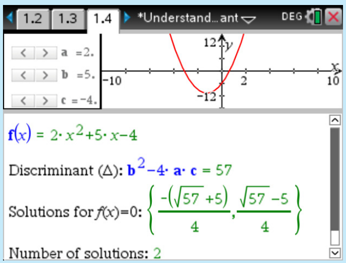
THE DISCRIMINANT: PEDAGOGICAL USE IN THE CLASSROOM

The two screenshots show page 1.2 and page 1.3 of a tns file called Understanding the Discriminant. Both pages have been constructed using the Notes application. These two pages contain explanatory text only and serve to introduce the interactive components of the tns file.





Page 1.4 of the tns file, shown in the screenshot, has been constructed using a split-page layout (**Documents > Page Layout > Select Layout > Layout 5**).



The top-left section of page 1.4 displays sliders (**Menu > Actions > Insert Slider**) for the parameters a, b and c. The top-right section of page 1.4 displays the graph of f (entered as $f1(x) = ax^2 + bx + c$). The three sliders and the Graphs application where the graph of f is displayed are dynamically linked. Hence, altering the slider values for the parameters a, b and c automatically changes the graph of f.

The bottom-section of page 1.4 is an interactive Notes application containing explanatory text and mathematical objects created with mathematical expression boxes. Changing the slider values for a, b and c also results in automatically updated outputs for the value of the discriminant (Δ), the exact solutions to the equation $ax^2 + bx + c = 0$ and the number of solutions to the equation $ax^2 + bx + c = 0$. From these dynamic outputs, students should be able to see that the quadratic formula can be expressed as $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$.

With teacher guidance, students should be able to devise and use appropriate sets of values for a, b and c to draw the following conclusions:

For $\Delta > 0$, the equation $ax^2 + bx + c = 0$ has two solutions, $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$, which are rational if Δ is a perfect square and irrational otherwise.

For $\Delta = 0$, the equation $ax^2 + bx + c = 0$ has exactly one solution, $x = \dfrac{-b}{2a}$.

For $\Delta < 0$, the equation $ax^2 + bx + c = 0$ has no real solutions.

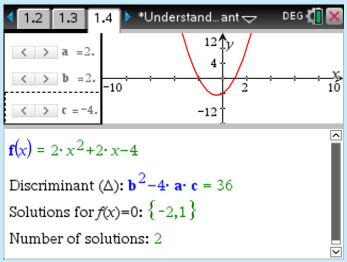
Since the solutions to the equation $ax^2 + bx + c = 0$ correspond to the x-axis intercepts of the graph of f, students should also be able to conclude that:

For $\Delta > 0$, the graph of f has two x-axis intercepts.

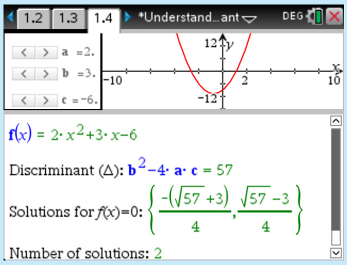
For $\Delta = 0$, the graph of f has one x-axis intercept i.e. the curve touches the x-axis at just one point.

For $\Delta < 0$, the graph of f has no x-axis intercepts.

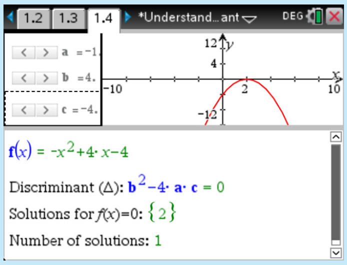
$\Delta > 0$ and Δ is a perfect square



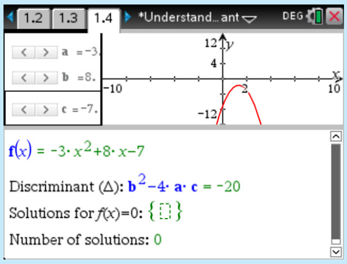
$\Delta > 0$ and Δ is not a perfect square



$\Delta = 0$



$\Delta < 0$



CONCLUSION

Widgets allow students the opportunity to create 'live' revision notes for use in examinations and hence answer exam questions accurately and efficiently. Interactive Notes pages consisting of explanatory text, diagrams, images and mathematical expression boxes afford opportunities for teachers and students to create powerful learning experiences. With such functionality, it is a great time to be teaching, learning and doing mathematics.

ONLINE COMMUNITIES

Michaela Epstein - Head of Learning, Maths Pathway


Meredith Payne
@pmerebearstyles
Following
I am so excited to announce I've accepted my first teaching job! 🌟 Now, I need first-year advice from my #PLN !
11:16 AM - 22 Apr 2019
25 Retweets 1,568 Likes
229 25 1.6K

It's your first teaching job. You're a bit excited by it, so you share the news. Before you know it, over a thousand strangers are warmly welcoming you, helping you to see that you are part of a broader community that has as its common thread your new profession.

In 2017, U.S. maths educator Dan Meyer set up a website that identified when new tweeters (i.e. people who tweet) used #MTBOS - the maths twitter blogosphere hashtag. As part of this site, Meyer and many others volunteered throughout each week to help publicly welcome the newcomers. The purpose? To build 'a community of math educators that meets online at all hours of the day. They trade support and resources.'

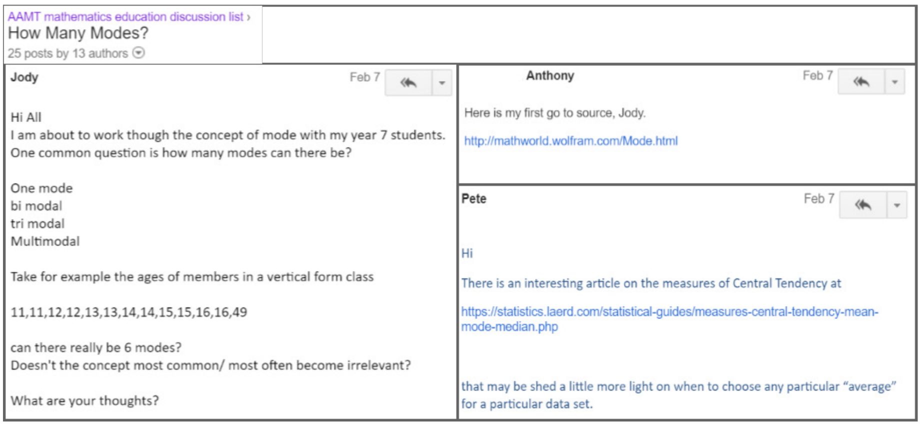
> community (noun): 1. A group of people living in the same place or having a particular characteristic in common. 2. The condition of sharing or having certain attitudes and interests in common. – Oxford Dictionary

It's easy to ignore what isn't physically right in front of you. When you're busy in the autopilot of a school term, looking ahead to the next meeting-free afternoon, upcoming weekend or school holiday is a more enticing thought than more work.

But scratch the surface and you'll see just how much can be gained from the online community of maths educators. This is a vibrant world of ideas, rich discussion and warm support. From enthusiastic new teachers to the extraordinarily experienced, the individuals within it call from all over the globe forming a community that I wholeheartedly encourage you to step into. By doing so, you are sure to benefit. Here I spell out three of the most common and most powerful benefits, along with some of the spaces this community calls home.

1. LEARN

Fear not getting tired of teaching the same old content or lost for stimulation!


AAMT mathematics education discussion list ›
How Many Modes?
25 posts by 13 authors

Jody — Feb 7
Hi All
I am about to work though the concept of mode with my year 7 students. One common question is how many modes can there be?

One mode
bi modal
tri modal
Multimodal

Take for example the ages of members in a vertical form class

11,11,12,12,13,13,14,14,15,15,16,16,49

can there really be 6 modes?
Doesn't the concept most common/ most often become irrelevant?

What are your thoughts?

Anthony — Feb 7
Here is my first go to source, Jody.
http://mathworld.wolfram.com/Mode.html

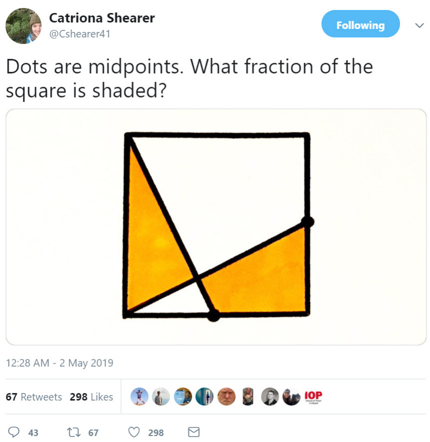
Pete — Feb 7
Hi
There is an interesting article on the measures of Central Tendency at
https://statistics.laerd.com/statistical-guides/measures-central-tendency-mean-mode-median.php

that may be shed a little more light on when to choose any particular "average" for a particular data set.

Need ideas for a lesson? Put a call out to the collective wisdom of the crowd. You're likely to get a range of ideas, including the thoroughly tried and tested.

And even without a specific content aim, you're bound to deepen your understanding of new corners of the curriculum. Around you there will be conversations on mathematics, pedagogy, cognitive development and more. Twitter and online forums give a space to weigh in with your knowledge, to ask questions or to just 'listen' to the voices of teachers, researchers and others. (See image above).

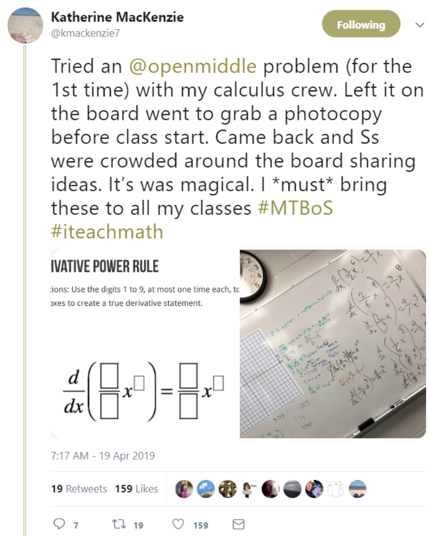
Then there's the maths that's just for the fun of it. Become fascinated by puzzles and problems that will draw you in. Shearer is one of my favourites, and just one of many educators who is designing and sharing original problems with a wow factor.


Catriona Shearer
@Cshearer41
Following
Dots are midpoints. What fraction of the square is shaded?
12:28 AM - 2 May 2019
67 Retweets 298 Likes
43 67 298

2. SHARE AND REFLECT

Help great ideas to be heard by passing them on to others. Just as you will learn from the great ideas of people you come across, passing an idea on upskills and saves time for others.

Sharing your own ideas - whether in blog form or in a discussion thread or tweet – will also help you to actively process what is in your head and can be a cathartic experience. By expressing your own ideas, you will make sense of what is important and what is worth remembering for the future.


Katherine MacKenzie
@kmackenzie7
Following
Tried an @openmiddle problem (for the 1st time) with my calculus crew. Left it on the board went to grab a photocopy before class start. Came back and Ss were crowded around the board sharing ideas. It's was magical. I *must* bring these to all my classes #MTBoS #iteachmath

7:17 AM - 19 Apr 2019
19 Retweets 159 Likes
7 19 159

3. CONNECT

The most likely outcome of joining a new community? Without a doubt, the connections you will make. Through the range of voices out there, you will find your online friends. These are people you trust and respect, and who will introduce you to likeminded others. They are your safety net.

Then there are the people whose perspectives will niggle away at you or whose ideas, at first glance, you completely disagree with. They are invaluable to this community – and to you. By seeking out ideas you respectfully don't like, you will see new perspectives and perhaps even change your mind in unexpected ways. At the very least, the act of engaging with different views helps us to build empathy.

So, where to go to form these connections?

- On Twitter: search for #MTBOS, #iteachmath or (for a non-maths focussed educational community) #aussieEd
- AAMTlist: A discussion forum hosted by the Australian Association of Maths Teachers. Get access via the AAMT website.
- https://blog.feedspot.com/high_school_math_blogs/: start with this site and go wild. In many blogs, there will be links on to others that are referenced or recommended.

If you are trying to figure out ideas for classes next week, get enthusiastic about teaching a topic or find out more about a new classroom routine, don't do it alone! There is a world of smart, responsive, funny and thought-provoking maths educators out there. By stepping into an online community you will learn loads from others, have opportunities to share and reflect, and form supportive and even lifelong connections.

This article was influenced by a session run by Michaela, Oliver Lovell (@ollie_lovell) and Amie Albrecht (@nomad_penguin) at the MAV18 Conference.

The topic was Strengthening the Maths Teacher Blogging Community. Michaela blogs at michaelaepstein.com.au and tweets at @mic_epstein.

REFERENCES

Meyer, D. 2017. 'How I Welcome Newcomers to Online Teacher Professional Development (a/k/a the #MTBoS) and How You Can Too', dy/dan. Accessed at https://blog.mrmeyer.com/2017/how-i-welcome-newcomers-to-online-teacher-professional-development-aka-the-mtbos-and-how-you-can-too/.

MAV BOARD 2019-2020

The MAV AGM was held on Tuesday 21 May 2019, at The Australian Council for Educational Research (ACER). The meeting began with a Special Resolution to adopt a new Constitution for the Association. In leading up to the adoption of the Constitution a thorough process had been ongoing for the previous 12 months. MAV engaged Mills Oakley Lawyers who are experienced in developing constitutions for the not-for-profit sector, to guide the Board and to ensure that the new constitution is legally sound, and will cater for the needs of MAV and its members into the future. You can read the Constitution on the MAV website, www.mav.vic.edu.au.

Under the new Constitution the MAV's Objects were refined and simplified to ensure that the focus of the association was clear and concise. The Object now read as follows:

The Objects of the Company are to assist and advance education in mathematics, and to promote the importance and value of mathematics in society. The Company will pursue these Objects through a range of activities and services including, but not limited to, a focus on:

i. supporting the development of a professional community of teachers, educators, and mathematicians, for the purpose of advancing mathematical education;

ii. promoting the importance and value of mathematics and the mathematical sciences in the wider community;

iii. providing professional learning for mathematics educators through events, conferences and publications;

iv. developing and promoting resources to facilitate the improvement of, and best practice in, teaching and learning of mathematics;

v. establishing connections with and between educational institutions, students, parents and carers, cultural institutions, government and government bodies, industry and the wider community for the advancement of mathematics and the mathematical sciences, mathematical education, and educational research and investigation;

vi. undertaking advocacy and representation, to uphold the profession to society and key stakeholders, and to support the improvement of the profession of mathematics education and mathematics teaching.

In line with MAV being a Company Limited by Guarantee, the previously named Council is now called the Board, and Councillors called Directors. Under the new Constitution an election was required to appoint the Elected Directors for the coming year. 11 Directors were elected by members via a ballot. Michaela Epstein continues as Immediate Past President. At the first meeting of the Board on June 4 2019, the Board selected from within the Board members the office bearers. The Board are very excited about leading the Association through the coming year. The MAV Board now consists of the following Directors:

- President, Michael O'Connor (AMSI)
- Vice President, Kate Copping (The University of Melbourne)
- Chair of Finance Committee, Juan Ospina Leon (South Oakleigh Secondary College)
- Immediate Past President, Michaela Epstein (Maths Pathway)
- Daniel Cloney (ACER)
- Claire Delaney (Lalor Secondary College)
- Ann Downton (Monash University)
- Trish Jelbart
- Peter Karakoussis (Scotch College)
- Allason McNamara (Trinity Grammar School)
- Kylie Slaney (Carey Baptist Grammar School)
- Max Stephens (The University of Melbourne)

You can read biographies of the Board on the MAV website under the 'About Us' section.

VCE REVISION LECTURES 2019

Each year the Mathematical Association of Victoria presents VCE revision lectures for students in the three VCE Mathematics subjects: Mathematical Methods, Specialist Mathematics and Further Mathematics. Students receive comprehensive notes and the lectures are delivered by highly qualified and experienced teachers, who are current VCAA exam assessors.

Make sure you reserve a seat by booking online early.

METRO

REGION	WHERE	WHEN
Eastern metro	Glen Waverley	Sunday 22 September
South East metro	Mt Eliza	Monday 23 September
Western metro	Taylors Lakes	Tuesday 24 September
Northern metro	Broadmeadows	Tuesday 1 October
Eastern metro	Ringwood	Sunday 13 October

REGIONAL

REGION	WHERE	WHEN
South East	Sale	Wednesday 25 September
South West	Horsham	Monday 30 September
North East	Wangaratta	Saturday 5 October

TIMES AND PRICES:

Mathematical Methods: 9am-1pm, $55
Further Mathematics: 1.30pm-5pm, $45
Specialist Mathematics: 1.30pm-4.30pm, $45

- SPECIAL -

BOOK TWO LECTURES FOR $85

STUDENTS ARE PROVIDED WITH DETAILED REVISION NOTES.

REGISTER NOW AT

www.mav.vic.edu.au/student-activities/vce-revision-lectures

THE MATHEMATICAL ASSOCIATION OF VICTORIA

TO REGISTER
www.mav.vic.edu.au/student-activities/vce-revision-lectures
OR CALL +61 3 9380 2399

RESOURCE REVIEW:
NUMERACY ACROSS THE CURRICULUM

Lee-Anne Pyke - Educational consultant, MAV

May is NAPLAN month across Australian schools and brings with it the anxieties and doubts associated with the NAPLAN assessment. Often as teachers we think and talk about the assessments as English and Mathematics assessment and forget that what is being assessed is Literacy and Numeracy.

Being numerate is not the same as being good at maths. Numeracy is not mathematics, but numeracy certainly encompasses mathematical knowledge.

Emerging from ten years of research, *Numeracy Across the Curriculum* provides an historical background to the inclusion of numeracy as a general capability in the Australian Curriculum, along with practical examples of learning experiences imbued with explicit numeracy demands applicable to all areas of the curriculum.

The Australian Curriculum identifies seven general capabilities of which one is Numeracy.

> *Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully (ACARA 2018).*

As with literacy, numeracy is a cross-curricular concern: however, numeracy has not been generally evident in practice.

Currently, some pre-service teachers undertake study units on numeracy, also referred to as quantative literacy, whilst many serving teachers have trouble defining numeracy as different to mathematics. In this book, the 21st Century Numeracy Model (Goos, Geiger and Dole 2014) is introduced and explained using real life examples to highlight the features of the model: mathematical knowledge, dispositions, tools and a critical orientation on solving problems in real-world contexts. The model elucidates a clear distinction between mathematics and numeracy.

Formatted in concise chapters with reflective tasks that enable the reader to independently construct their understanding as they work through the book. Ideally this book should be used by professional learning teams or as an individual professional development resource. The authors advocate for a whole school approach, to reinforce the cross-curricular relevance, and suggest models to develop numeracy practice throughout a school.

One example of a reflective task from the book engages teachers in a review of Numeracy within the Australian Curriculum in comparison the 21st Century Numeracy Model (p 81):

Review and Reflect 4.1

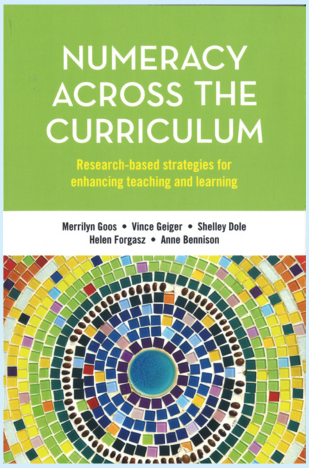
1. Visit the Numeracy section of the Australian Curriculum website and explore the organising elements for numeracy outlined there: 'Estimating and calculating with whole numbers'; Recognising and using patterns and relationships'; 'Using spatial reasoning'; 'Interpreting statistical information'; and 'Using measurement'.

2. Compare the elements of numeracy as proposed by the Australian Curriculum with elements of the 21st Century Numeracy Model introduced in Chapter 3.

3. Discuss the similarities and differences.

If we are to develop students' numeracy capabilities we must be providing rich learning experiences that include all the dimensions of numeracy: context, mathematical knowledge, dispositions, tools and critical orientation, along with appropriate pedagogy. The book discusses the principles of designing tasks and planning across the school to embed numeracy in whole school planning.

The 21st Century Numeracy Model can provide schools with a tool to rethink the NAPLAN data in the light of numeracy capabilities thereby embracing the whole Australian Curriculum for Mathematics rather than narrowly focussing on the content strands. This publication is an excellent tool to move thinking forward from teaching mathematics content to developing numerate students equipped and ready to take their place in society.



Numeracy Across the Curriculum is available from MAV's online shop, http://shop.mav.vic.edu.au.

THE VALUE OF A GOOD GAME

Cathy Epstein (Rodgers) - MAV Curriculum consultant, Numeracy Leader St Peter's East Bentleigh and St Paul's Bentleigh

Mathematical games are an intrinsic part of my mathematics teaching. They engage the children and are a great way to investigate or reinforce strategies or concepts.

One of the games I most frequently play is *Target 15 287* which can be found on the website www.nzmaths.co.nz. I have used this game or differentiated versions of it with a range of year levels, both playing with the whole class or in small groups. I play it when investigating place value, addition or subtraction or as a warm up.

To play you will need a 10 sided dice, 10 counters and a game board (Figure 1).

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000

Figure 1. Target 15 287 gameboard.

Roll	1	2	3	4	5	6	7	8	9	10
Number rolled	2	9	6	2	2	2	5	1	6	3
Choice	2000	900	6000	200	20	2	5000	1000	60	30
Total	2000	2900	8900	9100	9120	9122	14122	15122	15182	15212

Figure 2. Completed Target 15 287 gameboard after a game has been played.

Figure 3.

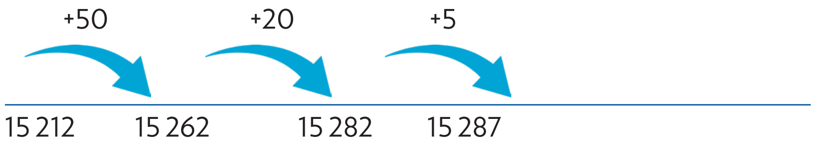


Figure 4.

The aim of the game is to be the closest to the target number 15 287 at the end of ten rolls of a dice by adding a selected value of the digit rolled. E.g. If you roll a 5 you can add 5, 50, 500 or 5000. The counters are placed on the digit choice for each roll. This keeps track of the value choices because once you have selected to use a roll of say 5 as 500, it cannot be selected again. As a class you may decide what to do if a zero is rolled. It can be zero added for that roll or roll again. If you go beyond 15 287 you bust!

I most commonly play *Target 15 287* by placing the students in pairs so one can keep track of the number choices and the other records the running total. This gives the pair the opportunity to discuss or justify choices for selecting the value of a number and ensures they agree that the rolling total is accurate. The teacher rolls for the whole class.

It is a good idea to get the students to draw up a table to keep track of the numbers rolled and the running total. Figure 2 shows an example of a table completed when a game was played.

Thoughtful discussion points could be.....

- Is there a strategy for winning?
- Is it better to go for the large numbers to total 15 000 first or should you aim to make 87 or 287?

What I love about this game is that it can be very easily differentiated depending on your focus or who you are working with. If you are playing this version with a class you can enable (or support students) by providing a calculator to check calculations, (ensuring they estimate first) or extend student thinking by limiting the digit value choices. There is also an extended version for senior grades called *Target 15.287* which can be found on the same website. This really challenges them to think carefully when adding or subtracting tenths, hundredths and thousandths especially if they have to rename. E.g 12.183 + 9 tenths (0.9)

If you are playing with a different level you can simply change the target by going up into the millions or down as low as say 120. In the latter case limit the number of rolls to 5, use a 6 sided dice and place value choices to units or tens.

Maybe choose a target of 20 and by rolling a dice labelled 1, 2, 3, 1, 2, 3 with the options that the students decide to add or subtract to get to closest to the target. Use a couple of tens frames for an enabling support or provide number charts to 120 or 1000 depending on the needs of the students and the target.

If you are investigating subtraction you can begin with 15 287 or the selected target and try to get closest to zero. To extend further you may move into negative numbers. Basically it is an ideal game to modify for any entry level by manipulating the target, dice, number of rolls and operations used.

I have also used this game with a focus group of students who were having trouble adding numbers involving zeros. E.g. One student had arrived at 12 970, rolled a 3 and needed to add 300 but was not sure how the number would change.

As a group we talked about which place value we were counting on from and counted on from 900 by 100 - 900, 1000, 1100, 1200, discussing the changes and then transferred this to 12 970, 13 070, 13 170, 13 270. We also investigated how we could mentally add this in our heads by partitioning 300 to round to the next 1000 and represented our thought process on an open number line (see Figure 3).

Upon completion of the game we have used the totalled numbers to explore how far away we all were from the target and shared different strategies for working this out (see Figure 4).

How else could I have worked this out? Some other things you could follow up with is ordering the totals or difference from greatest to least or asking children to pair up with someone to make a difference of around 50 or 100 etc.

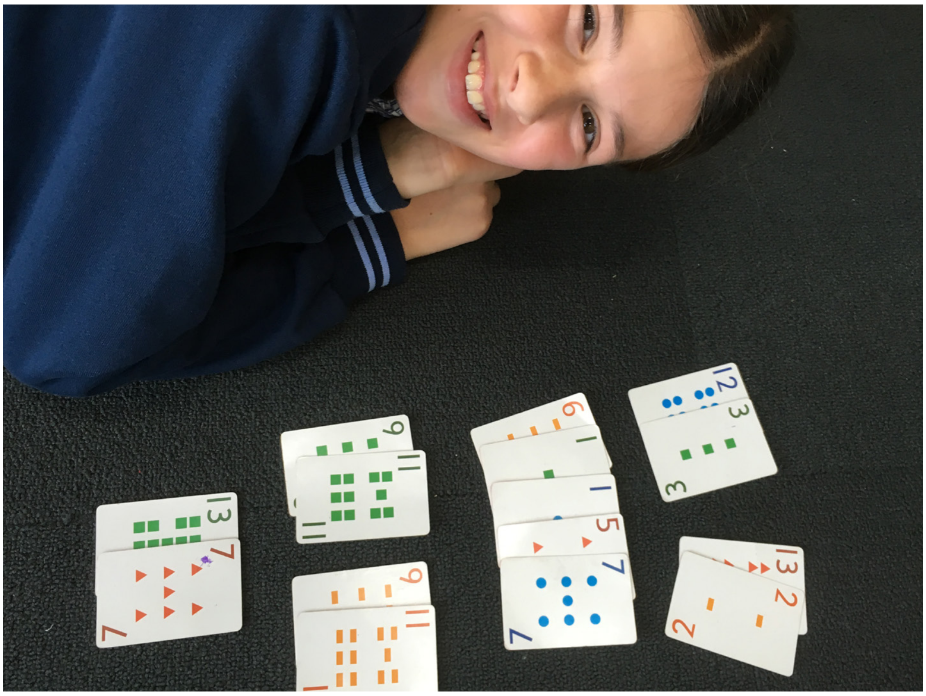
Target 15 287 is an excellent game which can be modified to explore a range of key ideas and concepts, and is easily differentiated to ensure all students can successfully participate.

ROWCO (ROWS AND COLUMNS)

Another valuable game I have played with class levels from Year 1 to Year 6 is *Rowco*, a game I first learned about from Paul Swan. I always play it with his student friendly cards, which are fabulous as they go from 0-13 instead of the picture cards and come in 4 different colours. To take it that bit further Paul also has sets of Rowco cards which include negative numbers. (Both can be purchased from MAV's online shop)

The game can be played one-on-one or in pairs where students can help each other to choose and calculate. To play, first deal the cards face up in an array. I usually begin with a 5 by 5 array. Choose one card to turn over. We call this the boss card.

One pair or one individual student chooses rows and the other chooses columns. The idea is to take turns to select the highest card in your row or column and replace it with the boss card. When someone has no cards left in their row or column and cannot have a go it is time to total your collection and the person or team with the highest total wins. Although the game itself is fun and there are some clever strategies when choosing cards, the real focus of the game is on the strategies the students use to efficiently add their collection. I always fishbowl and play the game with a selected child and we discuss as a class why I have grouped cards in particular ways to add efficiently. We then spread out my opposition's cards and discuss ways they can be grouped to add efficiently.



Madeleine (St Paul's Bentleigh) plays Rowco.

Most students sort their cards into friends or complements of 10 or 20, pairs or triples. By using Paul's cards you also have the option of pairing say 12 and 13 because you automatically know that is 25.

I was really excited recently to see some unexpected but efficient groupings to calculate.

Joshua grouped his: (13, 13, 3, 2, 9) (7, 13) (7, 13) (12, 2, 1) and proceeded to calculate 40, 60, 80 and 15 is 95.

While Madeleine grouped hers: (12, 3) (12, 3)(6, 1, 1, 5, 7)(9, 11)(9,11)(13, 7) to calculate 30, 50, 70, 90 and 20 equals a total of 110

Flynn grouped his (8, 8, 4) (11, 9)(13, 7) (11, 5, 4) (6) to calculate 20, 40, 60 and 26 is a total of 86.

Spotlighting and sharing strategies is very valuable as it may often highlight or present a way to calculate not thought of by a student who may now go and try that strategy.

Again this is a game that can be easily differentiated. Enabling tools could include tens frames to help create friends of 10 or a 120 number chart to help calculate the tally. Limiting the cards to 0-5, or 0-10 is also an option. To extend the task I have given the even cards a positive value and the odd cards a negative value, or doubled the value of a card.

There are also many rich activities you can investigate afterwards with the totals such as ordering, finding complements of particular numbers, benchmarking by asking for those closest to 50, 100, 150 or 200 and finding partners with common multiples etc.

TRY IT FOR YOURSELF

Mathematical games can be a fun and productive way to explore key ideas and topics or introduce and share efficient strategies to ultimately have our students working thoughtfully and confidently with numbers. Why not try one of these games and discover the value for yourself and your students!

The Term 3 edition of MAV's primary journal, *Prime Number*, is a bumper edition that focuses entirely on games and their application to mathematics. It's well worth checking out.

If you have a favourite game in your mathematics classroom, get in touch and tell us about it, office@mav.vic.edu.au.au.

COMPOUND INTEREST

Andrew Stewart

There are a number of quotes in circulation about compound interest, allegedly made by Albert Einstein.

> Compound interest is the eighth wonder of the world. He who understands it, earns it … he who doesn't … pays it.
>
> Compound interest is the most powerful force in the universe.
>
> Compound interest is the greatest mathematical discovery of all time.

Unfortunately, efforts to track the source of these comments have been unsuccessful, owing to the fact that they seem to have only appeared some twenty years after Einstein's death (*Einstein and Compound Interest*, 2011).

Showing Further Mathematics students the power of compounding interest often requires a more unusual approach to get the message across. The following examples could be used as part of the teaching notes, or better still, as part of a SAC Task to develop students' problem-solving skills.

The source of these examples is from a MAV Further Maths Trial Examination.

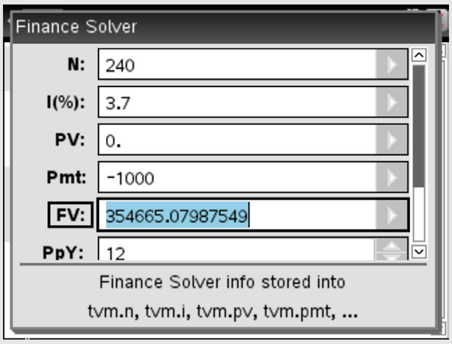
2018, Exam 1, Question 23

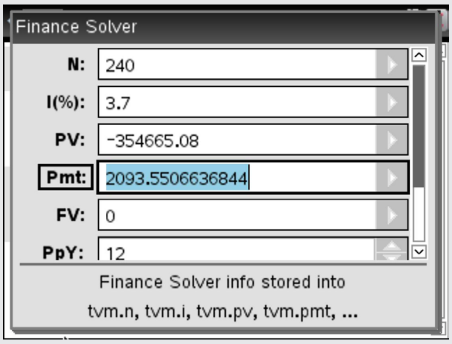
Travis starts with no money and over a 20-year period, he saves the same amount of money every month in an annuity investment. After the 20-year period, Travis invests the balance into an annuity that will make him equal monthly payments until it reaches a zero balance after another 20 years. The annuity investment and the annuity both compound monthly at 3.7% per annum. Which of the following statements is true?

A. Travis' annuity will pay the same amount as his monthly payment into the annuity investment
B. Travis will receive 3.7% more each month in his annuity than his payment into the investment
C. Travis will receive 7.4% more each month in his annuity than his payment into the investment
D. Travis will receive 37% more each month in his annuity than his payment into the investment
E. Travis' annuity will pay more than double his payment into the investment every month

The beauty of this question is that the amount that is invested does not matter – the answer can be found with any reasonable amount.

Let's start with $1000 as the monthly investment, and carry out the two calculations that will show what is happening. The two screen displays below show the amount accumulated in 20 years on the left ($354 665.08, correct to the nearest cent) and the monthly payment from the 20-year annuity on the right ($2093.55, correct to the nearest cent). Note that with $100 as the monthly investment, the amount accumulated in 20 years is $35 466.51, leading to a 20-year annuity payment of $209.36 – a similar result!





Clearly then, the correct answer is option E.

The annuity investment will continue to earn interest as the monthly payments are made, but because the annuity principal has a large value, a large amount of interest will be earned monthly, particularly in the early years, and hence a larger payment can be made to amortise the annuity in 20 years.

This concept can be turned into an investigation. At what interest rate, correct to two decimal places, will an investor receive at least a particular multiple of their annuity investment as an annuity? In both cases this is for a 20-year investment. The initial investment payment is not important, but an initial payment of at least $500 could be suggested to make the investment worthwhile. The multiples are easier to see with $1000 as the initial investment amount. The following table shows the interest rates that will give at least the annuity payment shown, for an initial investment of $1000.

Annuity	Rate
$2000	3.48%
$3000	5.51%
$4000	6.69%
$5000	8.08%
$6000	9.00%
$7000	9.77%

Obtaining a higher multiple of the initial investment amount clearly requires an interest rate that is much higher than that which is currently available, but what happens if the time is extended? The following table shows the rate, correct to two decimal places for a different number of years of investment/payback situations to return at least $3000 for a $1000 initial monthly investment.

Time	Rate
20	5.51%
21	5.25%
22	5.01%
23	4.79%
24	4.59%
25	4.41%
26	4.24%
27	4.08%
28	3.94%
29	3.8%
30	3.67%

We can see that the longer the investment/payback time, the lower the interest rate needed to achieve the required payment.

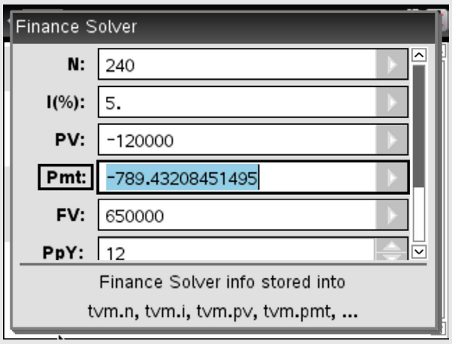
These questions have a great deal of flexibility, and a different combination of time and multiple could be given to each student in a class. (This also cuts down the copying!) To provide a little assistance, give a range within which the interest rate lies. For example, for a multiple of three paybacks over a 25-year investment/ annuity period, hint that the interest rate lies between 4.0% and 5.0% p.a.

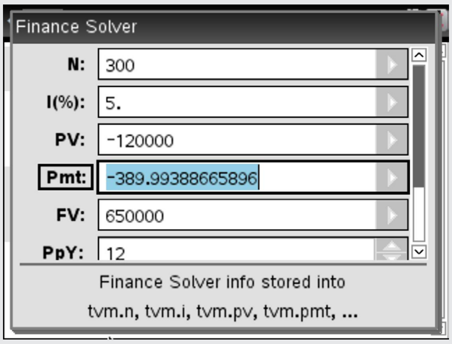
These types of problems will require extensive trial and error, and students should record all attempts, as these should help towards finding the required answer. The use of the phrase 'at least' should be interpreted for students as being the closest above that amount, correct to the required accuracy. This will overcome a student who claims 4.50% will do for a three-fold payment over 25 years. This gives a payment of $3053.74 compared to $3005.61 at 4.41%.

That same MAV Trial question inspired me to design the following SAC Task.

> Jill currently has an amount (over $100 000) in her superannuation account, but she wants to have at least a substantial amount (over $500 000) in the account when she plans to retire in twenty or twenty-five years.
>
> If her superannuation account is earning 4%-6 % interest per annum, compounded monthly, how much will Jill have to add each month, as a whole dollar amount, to at least meet her intended goal in 20 years or 25 years?
>
> Explain why there is such a significant difference in the monthly contribution required for these two time periods.

If the key numbers are carefully chosen, the 25-year contribution is about half the 20-year contribution. For example, as the screen displays show, starting from $120 000 with a target of $650 000 and earning interest at 5.0% p.a., the 20-year contribution will be $789, and the 25-year contribution will be $390. She ends up paying less because she is earning interest for a longer time (an extra five years), and she will be earning higher amounts of interest in those last five years.





To make this problem a little more difficult, students could be asked to find the interest rate, correct to two decimal places (within a specified range), for which the 20-year contributions will be double the 25-year contributions within a specified degree of accuracy, with particular starting and target amounts. Given that we may be working longer in the future, it could be worthwhile investigating longer time periods up to 35 years.

As an experienced Further Maths teacher, who has been involved in the subject since it started in 1994, I have often been asked where the inspiration comes for SAC activities/questions. The need for activities with an open-ended aspect in the latest Study Design has increased the difficulties of this process. Inspiration has come from my wide reading range, general news, sport, business and science, and from my interaction with colleagues, in particular my colleague Fiona LaTrobe, in writing StartPoints for many years, and more recently Trial Exams for the MAV.

Finding material for use in teaching notes or inspiration for SACs can be a challenging experience. Past examination papers, whether they are the MAV Trial Exams or the VCAA Exams, can be a fruitful source for this material.

With my retirement at the end of 2018, I now have more time to follow up on many of the ideas that I had had to put aside, and hope to include more ideas in future issues of *Common Denominator*.

REFERENCE

Einstein and Compound Interest. (n.d.) Retrieved February 22, 2019, from www.snopes.com/fact-check/compound-interest/. Snopes Media Group.

100 DAYS OF SCHOOL

Rosie Noone - Windsor Primary School and Jen Bowden - Mathematics Education Consultant, MAV

Jonathan and Jasper absorbed in 100 Days of School activities.

100 days of school has grown as a wonderful way for Foundation students, their teachers and families to celebrate an important milestone in a young student's educational journey.

The process of counting to the 100th day of school lends itself to rich learning activities and classroom routines that cover a range of mathematical concepts and skills.

Whilst the 100th day of school is a fun day full of celebration, the process of counting each school day is vitally important. Many Foundation learning spaces have a classroom displays in which students count the days of schools as a part of their morning routine. Each morning students:

- count the days, exposing them to ordinal number
- discuss the number before and the number after the current day
- physically represent the number either on a hundreds chart, collection of tens frames, bundling sticks, counters.

The anticipation leading up to the 100th day is evident in many schools with students proudly sharing with visitors what day they are up to and how many days they have to go to reach their 100 day milestone.

FINALLY, THE BIG DAY ARRIVES!

Schools celebrate in many different ways, children have dressed up as a school child from 100 years ago or in a creative fashion representing the number 100. Trays of cupcakes, collections of cars, beads, and an assortment of items are brought to school to show what 100 can look like. Both formal and informal measurement activities around the concept of 100 are explored. What is as long as 100 unifix cubes? What is a high as 100 cms? What might weigh 100 grams?

As enthusiasm for 100 days of school has grown, teachers have become more creative in their celebration and incorporating the concept of place value and the 'manyness' of 100 into their teaching and learning programs.

Activities involving creating patterns with 100 shapes, building environments with 100 lego blocks, completing activities in 100 seconds and rolling a dice 100 times are only just a few that have been explored.

The process can be used to teach concepts:

- counting leading towards addition
- concepts of before and after
- formal and informal measurement
- patterns
- shape and space
- time
- measurement

Whilst the 100th day of school is highly engaging for foundation students it also has a place across the school. Engaging older students in the celebration and asking extending question such as How many days have you been at school? What would school look like in 100 years time? What day of school do you think it will be on 15 December 2019? How many days will there be in 2019? How many school hours will there be in 2019?

Just like numbers don't stop at 100, the important process and routine of counting the days of school can continue past the 100th day. The rich and intentional learning that comes from a routine of counting the day of school, focusing on the day, date and time periods through the day continue to reinforce essential learnings.

100 DAYS AT WINDSOR PRIMARY SCHOOL

100 days of school has been an effective program in incorporating maths into our daily routine in an exciting and informal way. From the very first day of school, the program helps to reinforce, practice, and build student number sense to 100.

At the beginning of the year, our routine involves counting the days on a 100 chart, investigating the place value of each digit using straws and bundling, and writing the numbers on a whiteboard. The routine then progresses to more complex mathematical practices and can be linked to the various maths curriculum covered that week such as odd and even numbers, ordinal numbers, skip counting and mental computation strategies.

Prep students become incredibly excited as they edge closer to their 100th day of school. The day is celebrated by students coming to school in '100' themed dress ups (our costume of a 100 year old person was received well last year!), bringing or constructing a collection of 100 objects, fun maths games and craft activities and of course, a class party. Students have brought many amazing collections of 100 from a fairy garden, posters, cupcakes, necklaces and many more. The Year 5/6 students even get involved by helping the Preps read 100 books on our 100th day of school. The Prep students then present about their 100th day of school at assembly and enjoy sharing their achievement with the school.

100 days of school has had many positive impacts on the learning and development of my students and it is so pleasing to see the evolution of mathematical language and concepts understood and used and through the simple practice of 100 days of school.

Charting the days. This activity commences on day 1 of the school year.

The big day arrives! Windsor Primary School celebrated with numbered cupcakes.

How does your school incorporate mathematics into celebrations? We'd love you to share your story with our members in a future edition of *Common Denominator*.

Your story doesn't need to be word perfect, the editors can guide you through that process. If you are interested to learn more, email office@mav.vic.edu.au.

PUZZLES

Michael Nelson - Learning specialist, Drysdale Primary School

LOWER PRIMARY

Mike owns a pet store. He puts one parrot per cage, but has one bird too many. If he puts two parrots in each cage, he has one cage too many. How many cages and parrots does he have?

Recognise and represent division as grouping into equal sets and solve simple problems using these representations. (VCMNA109)

There are two ducks in front of two other ducks. There are two ducks behind two other ducks. There are two ducks beside two other ducks. How many ducks are there?

Describe position and movement. (VCMMG082)

MIDDLE PRIMARY

What is the next number in the sequence?

3, 4, 6, 10, 18, __

Describe, continue, and create number patterns resulting from performing addition or subtraction. (VCMNA138)

I bought an apple and a strawberry. The strawberry cost exactly $1 less than the apple. I paid $1.10 altogether. How much is the strawberry?

Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies. (VCMNA160)

UPPER PRIMARY

A snail is at the bottom of a 20 metre deep pit trying to get out. Every day the snail climbs 5 metres upwards but at night, it slides 4 metres back downwards. How many days does it take for the snail to get of the pit? (Adapted from Di Siemon)

Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction. (VCMNA192)

When my father was 31, I was 8. Now he is twice as old as me. How old am I?

Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers and make estimates for these computations. (VCMNA209)

CASIO

Prime Schools PLUS Program

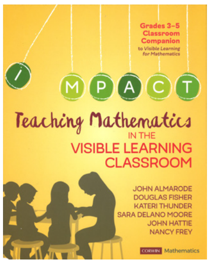
To become a Prime Schools PLUS school, it is as easy as following these 5 steps:

1. Head to: http://www.casio.edu.shriro.com.au
2. Click on the Prime Schools PLUS tab
3. Select ***"Join"***
4. Complete the form and provide a copy of your school's current book list or a letter signed by your Head of Mathematics (or equivalent), verifying that your school booklists one or more of the CASIO models listed overpage
5. We will do the rest!

MAVSHOP

http://shop.mav.vic.edu.au

MAV MEMBERS RECEIVE A 20% DISCOUNT ON ALL STOCK

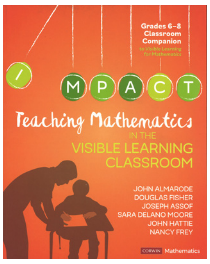


TEACHING MATHEMATICS IN THE VISIBLE LEARNING CLASSROOM

3–5

How do you generate that lightbulb 'aha' moment of understanding for your students? This book helps to answer that question by showing Visible Learning strategies in action in high-impact mathematics classrooms. Learn from teachers as they engage in the countless micro-decisions required to balance strategies, tasks, and assessments.

$46.77 (MEMBER)
$58.46 (NON MEMBER)

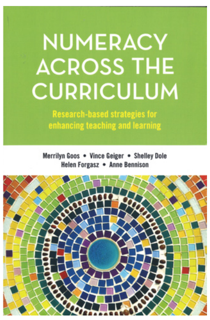


TEACHING MATHEMATICS IN THE VISIBLE LEARNING CLASSROOM

6–8

Select the right task, at the right time, for the right phase of learning. A decision-making matrix and grade-level examples help you leverage the most effective teaching practices at the most effective time to meet the surface, deep, and transfer learning needs of every student.

$45.08 (MEMBER)
$56.35 (NON MEMBER)



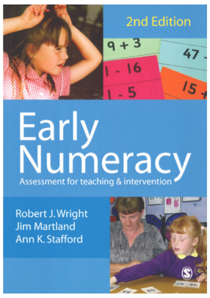
NUMERACY ACROSS THE CURRICULUM

F – VCAL

A guide for pre-service teachers on understanding numeracy and its application in both primary and secondary schooling.

Being numerate involves more than mastering basic mathematics. Numeracy connects the mathematics learned at school with out-of-school situations that require capabilities such as problem solving, critical judgment, and sense-making related to non-mathematical contexts. This book provides prospective and practising teachers with practical, research-based strategies for embedding numeracy across the primary and secondary school curriculum.

$42.24 (MEMBER)
$52.80 (NON MEMBER)



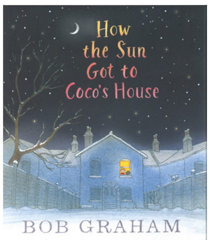
EARLY NUMERACY ASSESSMENT FOR TEACHING AND INTERVENTION (SECOND EDITION)

F–4

This book details the background and effectiveness of the highly regarded 'Mathematics Recovery' early intervention program, incorporating an interview-based approach to assessing young children's numerical knowledge and strategies. Developed in Australia and internationally regarded as a substantial contribution to the field, this work also formed the basis of the 'Count Me In Too' program.

The book includes six diagnostic interview schedules focussing on a range of aspects of early number, and sets out procedures for analysing the results of the assessment interviews together with a comprehensive framework (the Learning Framework in Number) for determining a child's strategies and documenting current levels of a child's knowledge.

$62.71 (MEMBER)
$78.39 (NON MEMBER)



HOW THE SUN GOT TO COCO'S HOUSE

F–6

This is an enchanting story about the sun, and how it makes its journey from the far side of the world to the home of one small girl. The sun rises up behind a snowy peak and casts its mellow dawn light for the wandering polar bears. It skims across the icy water, touching a fisherman's hat and catching for a moment in the eye of a whale. It beams through the trees of frozen forests and makes shadows in a little girl's footsteps before gliding over cities, darting down lanes and waiting patiently for an old lady to open her window. The sun races through the countryside, greeting snow cats and bears. High over a desert it meets the rain in a halo of colours. The sun leaps whole countries, chasing the night, before bursting at last in a fanfare of warm golden light through Coco's window!

$21.11 (MEMBER)
$26.39 (NON MEMBER)

THE MATHEMATICAL ASSOCIATION OF VICTORIA

TO ORDER
http://shop.mav.vic.edu.au
OR CALL +61 3 9380 2399

Prices are subject to change.